Topic: Algebraic Manipulation Worksheet 1

Things to note:
1. In simplifying algebraic fractions, multiply or divide the numerator and denominator by the same non-zero number or expression.
2. Factorise the numerator and denominator to see if there are any common factors.
3. Never cancel individual terms of the numerator and the denominator before factorisation is done.
4. Apply rules of indices carefully.

Useful Shortcuts (Identities)

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<table>
<thead>
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<tbody>
<tr>
<td>1.</td>
<td>( a - b = -(b - a) )</td>
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<tr>
<td>2.</td>
<td>( -a - b = -(a + b) )</td>
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<tr>
<td>3.</td>
<td>((a + b)(a - b) = a^2 - b^2)</td>
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<tr>
<td>4.</td>
<td>((a + b)^2 = a^2 + 2ab + b^2)</td>
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<td>5.</td>
<td>((a - b)^2 = a^2 - 2ab + b^2)</td>
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(1) Simplifying Algebraic Fractions
This concept is similar to the arithmetical method of reducing numeral fractions to its lowest terms.

Example 1: Simplify \(\frac{3x^3yz^8}{27xy^3z^4}\)
Solution:

Example 2: Simplify \(\frac{9(x + y)^{n+1}}{24(x + y)^{n+3}}\)
Solution:

Example 3: Simplify \(\frac{x^2 + xy - xz - yz}{x^2 + xy + xz + yz}\)
Solution:

Example 4: Simplify \(\frac{2a^2 - 17a + 21}{-a^2 + 6a + 7}\)
Solution:
(2) Multiplying and Dividing Algebraic Fractions

How do you multiply fractions?

\[
\frac{18 \times 35}{49} = \frac{9 \times 2}{7 \times 7} \times \frac{5 \times 7}{3 \times 9} = \frac{2 \times 5}{7 \times 3} = \frac{10}{21}
\]

What did we do to 18, 35, 49 and 27? _________________________

We apply the same concept to algebraic fractions!

**Example 1:** Simplify \( \frac{x^2 + xy}{x^2 - xy} \times \frac{xy^2 + y^3}{x^3 + x^2 y} \)

Solution:

**Example 2:** Simplify \( \frac{x - y}{b - a} \times \frac{a - b}{y - x} \)

Solution:

**Things to note:**

1. To divide by a fraction, multiply by its reciprocal.
2. Simplify the result by cancelling common factors.

**Useful Shortcut**

\[
\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}
\]

(3) Addition and Subtraction of Algebraic Fractions

How do you add the following fractions: \( \frac{2}{5} + \frac{5}{12} \)?

We will use the same idea to add algebraic fractions.

**Example 1:** Simplify \( \frac{y + 2}{3} - \frac{3 - y}{5} \)

Solution:
Example 2: Simplify \[
\frac{2x}{x-1} - \frac{3x-1}{x+2}
\]
Solution:

Example 3: Simplify \[
\frac{x}{x+1} - \frac{2}{1-x^2}
\]
Solution:

(4) Solve equations involving algebraic fractions

Example 1: Solve the equation \[
\frac{2x - 3}{3} = \frac{x - 2}{5}
\]
Solution:

Example 2: Solve the equation \[
\frac{3x - 1}{5} = \frac{2x - 5}{2} = 1
\]
Solution:

Example 3: Solve the equation \[
\frac{2}{a} - 3 = 2a
\]
Solution: