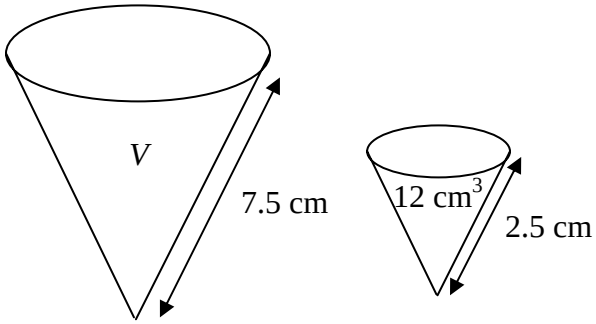
**Examples:**

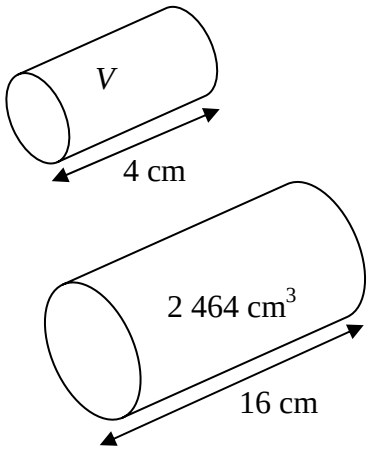
1 Find the unknown volume, $V$, of each of the following solids.




$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$

Ans: $$\frac{V_1}{12} = \left(\frac{7.5}{2.5}\right)^3$$

$$V_1 = 324 \text{ cm}^3$$

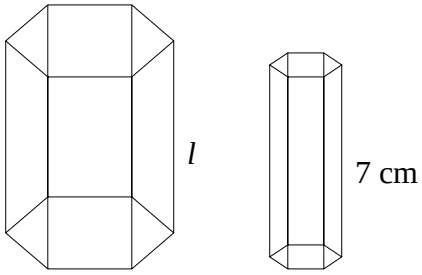


$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$

Ans: $\frac{V_1}{2464} = \left(\frac{4}{16}\right)^3$

$$V_1 = 38.5 \text{ cm}^3$$

2 Find the unknown length, $l$, in each of the following pairs of similar objects.

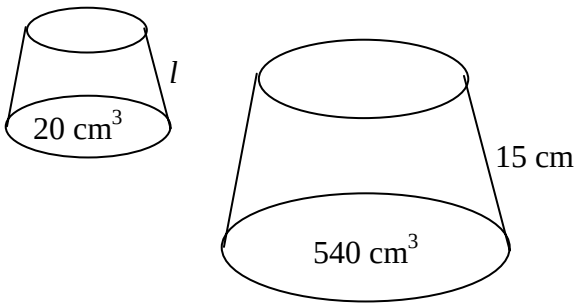



270 cm$^3$ 10 cm$^3$

$$\frac{270}{10} = \left(\frac{l}{7}\right)^3$$

Ans: $\sqrt[3]{27} = \frac{l}{7}$

$$l = 21 \text{ cm}$$



$$\frac{20}{540} = \left(\frac{l}{15}\right)^3$$

Ans: $\sqrt[3]{\frac{1}{27}} = \frac{l}{15}$

$$l = 5 \text{ cm}$$

3 The areas of the bases of 2 similar cones are in the ratio 9:16.
(a) Find the ratio of the heights of the cones.
(b) Given that the volume of the larger cone is 448 $cm^3$, find the volume of the smaller cone.

(a) $\left(\frac{h_1}{h_2}\right)^2 = \frac{9}{16}$

$$\frac{h_1}{h_2} = \frac{3}{4}$$

(b) $\left(\frac{h_1}{h_2}\right)^3 = \frac{V}{448}$

$$\left(\frac{3}{4}\right)^3 = \frac{V}{448}$$

$$V = 189 cm^3$$

**Exercises:**

Q1 Find the ratio of (a) the surface areas (b) the volumes of

(i) 2 similar solid cylinders of circumferences 10 cm and 8 cm;
(ii) 2 similar solid cones of heights 9 cm and 12 cm;
(iii) 2 spheres of radii 4 cm and 6 cm.

(i) $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 = \left(\frac{10}{8}\right)^2 = \frac{25}{16}$

$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3 = \left(\frac{10}{8}\right)^3 = \frac{125}{64}$

(ii)
$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 \qquad \frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$
$$= \left(\frac{9}{12}\right)^2 \qquad = \left(\frac{9}{12}\right)^3$$
$$= \frac{9}{16} \qquad = \frac{27}{64}$$

(iii)
$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 \qquad \frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$
$$= \left(\frac{4}{6}\right)^2 \qquad = \left(\frac{4}{6}\right)^3$$
$$= \frac{4}{9} \qquad = \frac{8}{27}$$

Q2 The volume of one sphere is 4 times the volume of a second sphere. If the radius of the smaller sphere is 3 cm, what is the radius of the larger sphere?

Ans:
$$V_1 = 4V_2$$
$$\frac{V_1}{V_2} = \left(\frac{r_2}{3}\right)^3$$
$$\frac{4V_2}{V_2} = \left(\frac{r_2}{3}\right)^3$$
$$4 = \frac{r_2^3}{27}$$
$$r_2 = 4.7622...$$
$$\approx 4.76cm$$

Q3 The masses of 2 similar marble toys of the same density are 8.58 kg and 4.29 kg. If the first toy is 12.94 cm high, what is the height of the second toy?

$Mass = Volume \times Density$ → Mass and Volume have the same ratio.

$$\frac{M_1}{M_2} = \left(\frac{h_1}{h_2}\right)^3$$

$$\frac{8.58}{4.29} = \left(\frac{12.94}{h_2}\right)^3$$

$$\left(\frac{12.94}{h_2}\right)^3 = 2$$

$$h_2 = 10.27048...$$

$$\approx 10.3cm$$

Q4 The height of the mast of the model of a boat is 14 cm, and the height of the mast of the actual boat is 2.1m.

(a) Given that the length of the model boat is 160 cm, calculate the length of the actual boat in cm.

$$\frac{14}{210} = \frac{160}{l_2}$$

$$l_2 = 2400cm$$

(b) If it costs $4 to paint the model, what will it cost to paint the actual boat?

$$\left(\frac{14}{210}\right)^2 = \frac{4}{A_2}$$

$$A_2 = \$900$$

Q5 Alan buys two bottles of milk at Golden Storage Supermarket. The bottles are geometrically similar to each other. The base diameter of one bottle is 8 cm and the base radius of the other bottle is 3 cm.

(a) Calculate the height of the small bottle if the height of the larger bottle is 28 cm.

$$\frac{4}{3} = \frac{28}{h_2}$$

$$h_2 = 21cm$$

(b) Calculate the ratio of the volume of large bottle to the volume of small bottle.

$$\left(\frac{4}{3}\right)^3 = \frac{64}{27}$$

(c) The cost of the small bottle of milk is \$2.70. Calculate the cost of the large bottle of milk .

$$\left(\frac{4}{3}\right)^3 = \frac{C_2}{2.7}$$
$$C_2 = \$6.40$$

Q6 (a) Two similar solids have surface areas of 160 $cm^2$ and 78.4 $cm^2$ respectively.

(i) Write down the ratio of the heights of the solids, in the form 1 : *n*, where *n* is a constant.

$$\sqrt{\left(\frac{78.4}{160}\right)} = \frac{7}{10}$$
$$7:10$$
$$1:1.43(3sf)$$

(ii) If the height of the smaller solid is 6 cm, find the height of the larger solid.

$$\frac{7}{10} = \frac{6}{h}$$
$$h = 8.5714..$$
$$\approx 8.57cm$$

(iii) Given that the volume of the larger solid is 650 $cm^3$, calculate the volume of the smaller solid.

$$\left(\frac{7}{10}\right)^3 = \frac{V}{650}$$
$$V = 222.95cm^3$$

(b) Two similar cones have surface areas $A_1$ and $A_2$ such that $A_1 = 169A_2$. If their volumes, $V_1$ and $V_2$ respectively, are such that $V_1 = kV_2$, write down the value of $k$.

$$A_1 = 169A_2$$

$$\frac{A_1}{A_2} = \frac{169}{1}$$

$$\frac{l_1}{l_2} = \frac{13}{1}$$

$$\frac{V_1}{V_2} = \frac{2197}{1}$$

$$V_1 = 2197V_2$$

$k = 2197$