

Revision Paper 3 – Further Indices Solutions

Q1 Evaluate the following.

$$\begin{aligned} \text{(a)} \quad 32^{-0.6} \\ &= \frac{1}{32^{\frac{3}{5}}} \\ &= \frac{1}{8} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 256^{(-3)(-2)} \\ &= 256^{\frac{1}{9}} \end{aligned}$$

*Note: $256^{(-3)(-2)} = 256^6$. Size and Position matters!

Q2 Simplify the following,

$$\begin{aligned} \text{(a)} \quad 12^{n-1} \times 16^{2n+1} \div 36^{1-n} \\ &= 2^{n-1} \times 2^{n-1} \times 3^{n-1} \times 2^{8n+4} \div (2^{2-2n} \times 3^{2-2n}) \\ &= \frac{2^{10n+2} \times 3^{n-1}}{2^{2-2n} \times 3^{2-2n}} \\ &= 2^{12n} \times 3^{3n-3} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \frac{3^{3n+2} \div 3^n}{3^{n+2} - 3^n} \\ &= \frac{3^{2n+2}}{3^n(3^2 - 1)} \\ &= \frac{3^{n+2}}{8} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad 6x^2 \div \left(\frac{2}{3}xy^3\right)^{-2} \\ &= 6x^2 \div \left(\frac{3}{2xy^3}\right)^2 \\ &= 6x^2 \div \left(\frac{9}{4x^2y^6}\right) \\ &= 6x^2 \times \left(\frac{4x^2y^6}{9}\right) \\ &= \frac{8}{3}x^4y^6 \end{aligned}$$

Q3 Solve the equation $7^{x-2} \div 7^{3x-2} = \frac{1}{343}$.

$$\begin{aligned} 7^{x-2} \div 7^{3x-2} &= \frac{1}{343} \\ 7^{-2x} &= 7^{-3} \\ -2x &= -3 \\ x &= 1.5 \end{aligned}$$

Q4 Find the value of k if $\sqrt{x\sqrt{x\sqrt{x}}} = x^{2k}$

$$\begin{aligned}
\sqrt{x\sqrt{x\sqrt{x}}} &= \sqrt{x\sqrt{xx^{\frac{1}{2}}}} \\
&= \sqrt{x\sqrt{x^{\frac{3}{2}}}} \\
&= \sqrt{x\left(x^{\frac{3}{2}}\right)^{\frac{1}{2}}} \\
&= \sqrt{x^{\frac{7}{4}}} \\
&= x^{\frac{7}{4} \times \frac{1}{2}}
\end{aligned}$$

$$x^{\frac{7}{8}} = x^{2k}$$

$$2k = \frac{7}{8}$$

$$k = \frac{7}{16}$$

Q5 Simplify $\frac{3^n \times 12^{2n+1} \div 16^2}{3^{2n} - 3^{2n+1}}$ and leave your answer in index form.

$$\begin{aligned}
&= \frac{3^n \times 4^{2n+1} \times 3^{2n+1} \div 4^4}{3^{2n}(1-3^1)} \\
&= \frac{3^{3n+1} \times 4^{2n-3}}{3^{2n}(-2)} \\
&= -\frac{3^{n+1} \times 2^{4n-6}}{2} \\
&= -3^{n+1} \times 2^{4n-7}
\end{aligned}$$

Q6 Given that $x^y = z$, find

(a) z when $x = 9$ and $y = \frac{1}{2}$.

$$\begin{aligned}
9^{\frac{1}{2}} &= z \\
z &= 3
\end{aligned}$$

(b) z when $x = 64$ and $y = -\frac{1}{2}$.

$$\begin{aligned}
64^{-\frac{1}{2}} &= z \\
z &= \frac{1}{64^{\frac{1}{2}}} \\
z &= \frac{1}{8}
\end{aligned}$$

(c) x when $y = -\frac{1}{2}$ and $z = 4$.

$$\begin{aligned}
x^{-\frac{1}{2}} &= 4 \\
x &= 4^{-2} \\
x &= \frac{1}{16}
\end{aligned}$$

Q7 Evaluate without using tables or calculators:

$$\begin{aligned}
 \text{(a)} \quad & \left(\frac{81}{256}\right)^{-\frac{3}{4}} \\
 &= \left(\frac{256}{81}\right)^{\frac{3}{4}} \\
 &= \left(\frac{4}{3}\right)^3 \\
 &= \frac{64}{27} \\
 \text{(b)} \quad & \left(-\frac{729}{1331}\right)^{-\frac{1}{3}} \\
 &= \left(-\frac{1331}{729}\right)^{\frac{1}{3}} \\
 &= -\frac{11}{9} \\
 \text{(c)} \quad & 49^{0.5} \div \left(\frac{1}{32}\right)^{0.4} \\
 &= 7 \div \left(\frac{1}{32}\right)^{\frac{2}{5}} \\
 &= 7 \div \frac{1}{4} \\
 &= 28
 \end{aligned}$$

Q8 Evaluate the following, given that $x + y = 3$ and $xy = 4$.

$$\begin{aligned}
 \text{(a)} \quad & (x^{-1} + y^{-1})^{-1} \\
 &= \frac{1}{x^{-1} + y^{-1}} \\
 &= \frac{1}{\frac{1}{x} + \frac{1}{y}} \\
 &= \frac{1}{\frac{y+x}{xy}} \\
 &= \frac{xy}{x+y} \\
 &= \frac{4}{3} \\
 \text{(b)} \quad & \left(\frac{1}{(x+y)^{-1}}\right)^{-1} \\
 &= \frac{(x+y)^{-1}}{1} \\
 &= \frac{1}{(x+y)} \\
 &= \frac{1}{3}
 \end{aligned}$$

Q9 Simplify the following, leaving your answers in positive indices only

$$\begin{aligned}
 \text{(a)} \quad & \frac{(a^3)^{-2}b^5}{4a^2b^{-1}} \\
 &= \frac{a^{-6}b^5}{4a^2b^{-1}} \\
 &= \frac{1}{4}a^{-8}b^6 \\
 &= \frac{b^6}{4a^8} \\
 \text{(b)} \quad & \left(\frac{-2}{3}\right)^3 + \sqrt[3]{-27} \\
 &= -\frac{8}{27} + (-3) \\
 &= -3\frac{8}{27}
 \end{aligned}$$

Q10 Solve for the value of x , $8^{x-1} = 16^{x+3}$.

$$2^{3x-3} = 2^{4x+12}$$

$$3x-3 = 4x+12$$

$$x = -15$$