Revision Paper 14: Algebra II

- Quadratic Equations
- Indices
- Simultaneous Equations
- Linear Inequalities

Q1 Solve $\frac{2}{x+2}-\frac{x}{x+3}=1$. $x=0,-2\frac{1}{2}$

Q2 Solve $(x-1)(x+2)=4$. $x=2\ or\ -3$

Q3 Solve $(2-2x)^2=14$. $x=2.87$ or -0.871

Q4 Solve $2(x+1)^2-6=0$ $x=\pm\sqrt{3}-1$

Q5 Solve the equation $x^2-3x+2=0$.

Hence solve the equation $y^4-3y^2+2=0$. $x=2$ or $x=1$, $y=\pm\sqrt{2}$ or ±1

Q6 Solve the following equations.

(a) $\frac{3}{e}=\frac{4e}{27}$ ±4.5

(b) $4x^2=7x$ $x=0$ or $x=1\frac{3}{4}$

(c) $3x^2+13x-10=0$ $x=\frac{2}{3}$ or $x=-5$

(d) $(x+2)(x-3)=(x+2)$ $x=-2$ or $x=4$

Q7 Solve the equation $4x^2-5=x(x-14)$. $\frac{1}{3},-5$

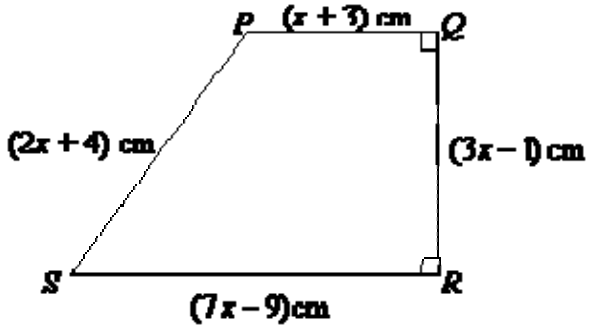
Q8 Solve $\left(4p+2\frac{3}{4}\right)^2=0.08$. $p=-0.617$ or -0.758

Q9 The solutions of the equation $x^2+ax+b=0$ are 5 and -6. Find the values of a and b. $a=1, b=-30$

Q10 If $y=x^2-4x-44$, $y=148$ find the values of x. $x=-12\ or\ 16$

Q11 PQRS is a trapezium in which $\angle PQR=\angle QRS=90°$.

$PQ = (x + 3)$ cm, $QR = (3x-1)$ cm, $RS = (7x-9)$ cm and $PS = (2x + 4)$ cm.



(a) Given that the area of the trapezium is 72 cm^2, form an equation in x and show that it reduces to $12x^2 - 13x - 69 = 0$.

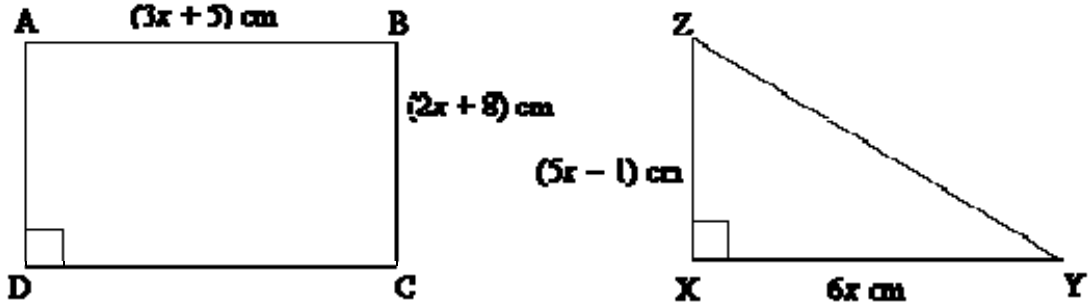
(b) Solve the equation $12x^2 - 13x - 69 = 0$ $3 \text{ or } -1\frac{11}{12}$

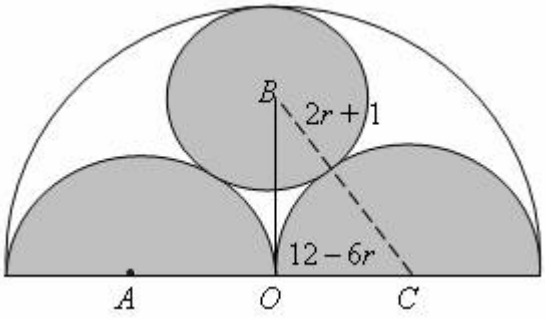
(c) Write down, in terms of x, the length of the perimeter of the trapezium.
Hence, find the perimeter. $(13x - 3)$ cm, 36 cm

Q12 ABCD is a rectangle in which AB = $(3x + 5)$ cm and BC = $(2x + 8)$ cm. XYZ is a right-angled triangle in which XY = $6x$ cm and XZ = $(5x - 1)$ cm. Given that the area of the rectangle ABCD is equal to the area of the triangle XYZ.

(a) Form an equation in x and show that it reduces to $9x^2 - 37x - 40 = 0$.

(b) Solve this equation and find the length of XZ. (b) XZ = 24 cm





Q13
In the diagram, the circle with centre at B has a radius $(2r + 1)$ cm. The semicircle with centre at A and the semicircle with centre at C are identical and each has a radius $(12 - 6r)$ cm. O is the centre of the largest semicircle.

(a) Write down an expression, in terms of r, for

(i) BC, $13 - 4r$

(ii) BO. $23 - 14r$ or $\sqrt{-20r^2 + 40r + 25}$

(b) Form an equation in r and show that it reduces to $18r^2 - 57r + 42 = 0$.

(c) Solve the equation to find the possible value of r. 2 (NA) or $1\frac{1}{6}$ cm

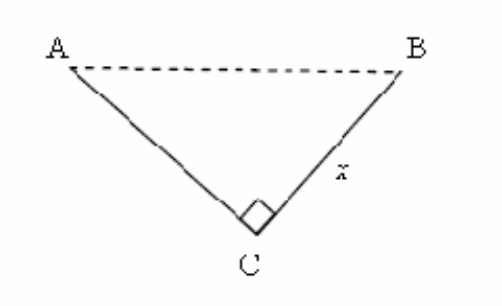
(d) Find the area of the unshaded region. 43.6 cm^2

Q14 In an experiment, the ends of a string of length 14 meters are attached to two points, A and B. A weight is fixed to a point C on the string as shown in the diagram below.
The two parts of the string, AC and BC are both straight. The length of BC is x meters.

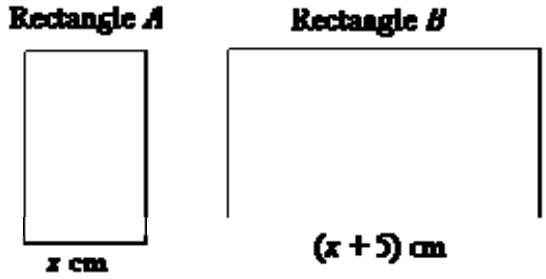
(i) Write down, in term of x , the length of AC. $AC = 14 - x$

(ii) Given that AB = 10 meters and $\angle ACB = 90°$, form an equation involving x , and show that it simplifies to $x^2 - 14x + 48 = 0$

(iii) Solve the equation $x^2 - 14x + 48 = 0$ $x = 8, x = 6$



Q15



The two rectangles shown above have an area of 20 cm^2 each.
The length of Rectangle A is x cm and the length of Rectangle B is $(x + 5)$ cm.

(a) (i) Find, in terms of x, an expression for the width of Rectangle A. $\left(\frac{20}{x}\right)$ cm

(ii) Find, in terms of x, an expression for the width of Rectangle B. $\left(\frac{20}{x+5}\right)$ cm

(b) Given that the width of Rectangle A is $\frac{1}{5}$ cm greater than width of Rectangle B, form an equation in x and show that it reduces to $x^2 + 5x - 500 = 0$.

(c) Solve the equation and find the value of x. $x = 20$

(d) Hence, find the width of Rectangle B. $\frac{20}{20+5}$

Q16 (a) Marble square tiles of side x centimeters, are to be stuck to a wall so that they fill a rectangular space 240 cm by 160 cm. Some of the tiles are shown in Figure *A*.

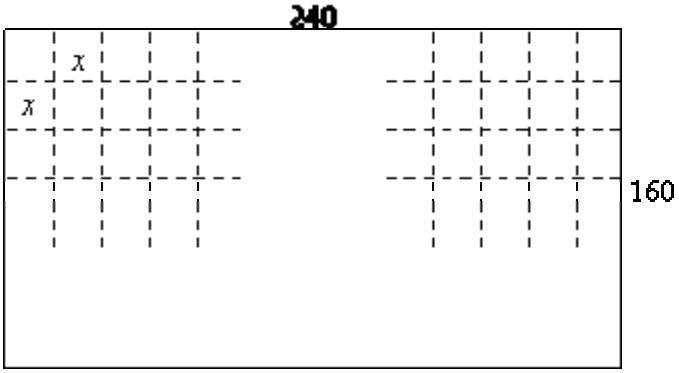


Figure *A*

(i) Write down an expression, in terms of x, for the number of tiles that will fit across the top row. $\frac{240}{x}$

(ii) Given that 600 tiles are required to fill the whole space, calculate x. $x = 8$cm

(b) Figure *B* shows another rectangular space which is 240 cm by 160 cm. This is to have one row of rectangular tiles stuck inside each edge so that they cover the unshaded area only.

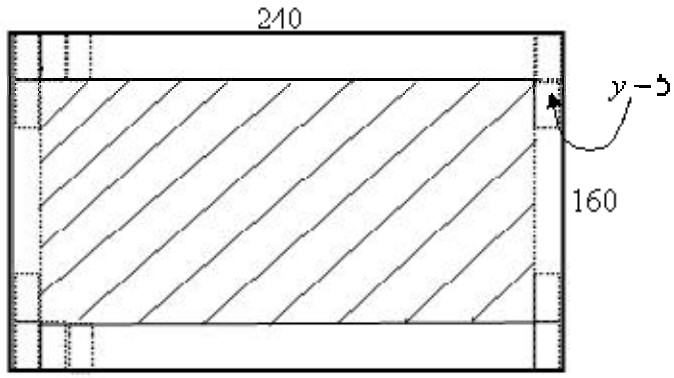


Figure *B*

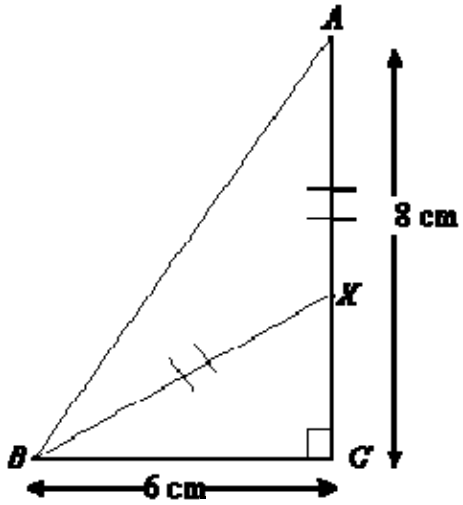
The tiles measure y centimeters by $(y-5)$ center. Each tile is placed so that its longer side is vertical. Some of the tiles are shown in the diagram.

(i) Write down an expression, in terms of y, for the number of tiles that will fit across the top row. $\frac{240}{y-5}$

(ii) Given that 44 tiles are required to fill the whole unshaded area, form an equation and show that it reduces to $3y^2 - 65y + 100 = 0$.

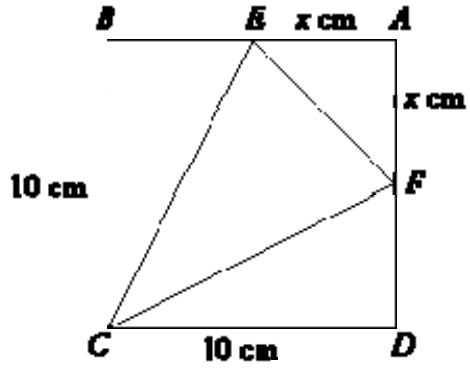
(iii) Solve this equation and hence find the length of the shorter side of a tile. 15 cm

Q17



The diagram (not drawn to scale) shows a right angled triangle *ABC*. *AC* is 8 cm and *BC* is 6 cm. Point *X* on *AC* is equidistant from *A* and *B*. Find *CX*. 1.75 cm

Q18



In the figure, *ABCD* is a square of side 10 cm. If $AE = AF = x$ cm and the area of ΔCEF is 32 cm^2,

(a) write down an equation in x and show that it reduces to $x^2 - 20x + 64 = 0$.

(b) solve the equation $x^2 - 20x + 64 = 0$. $x = 16$ or $x = 4$

(c) Explain why you would reject one of the two solutions.

$x = 16$ is rejected because x should be less than 10 cm which is the length of the square.

Q19 Simplify

(a) $\left(\frac{1}{3}xy^2\right)^2$, $\frac{1}{9}x^2y^4$

(b) $\left(\frac{3x}{p}\right)^2 + \left(\frac{qp^2}{2}\right)^{-3}$. $\frac{9x^5p^4}{8}$

Q20 Simplify and express $\frac{30x^3y}{7x^3y^2} + \frac{5xy^3}{21x^4y}$ as a single fraction. $\frac{18}{y^3}$

Q21 Simplify $\left(5s^{\frac{5}{2}}t^{-1}\right)^2 \div \left(s^{-\frac{1}{3}}\right)^{\frac{1}{2}}$ and express your answer in positive index form. $\frac{25s}{t^2}$

Q22 Given $\frac{t^3 \times t^3}{\sqrt{t}} = t^n$. Find the value of n. 4.5

Q23 Simplify $\frac{(a^3b^{-1}c)^3}{\sqrt{4a^6}}$, giving your answer in positive indices only. $\frac{a^6c^3}{2b^3}$

Q24 Simplify $\left(\frac{5}{2y^2}\right)^{-3}$. $\frac{8y^6}{125}$

Q25 Simplify $\left(\frac{x^2y^{-3}}{x^{-4}y^2}\right)^2 \times \left(\frac{x^{-2}y^{-1}}{x^5y^3}\right)^{-3}$, giving your answer in positive indices. x^{20}

Q26 Simplify $(7x^2y)^0 \div (4x^3)^{-1}$. $4x^3$

Q27 Simplify

(a) $\left(\frac{1}{3x}\right)^3 - (81y^4)^{-\frac{1}{2}}$ $\frac{y^2}{3x^3}$

(b) $\left(\sqrt{\frac{4}{9x^6}}\right)^{-1} + \left(\frac{x^5}{32}\right)^{0.8}$ $\frac{25x^4}{16}$

Q28 Given that $\left(81^{\frac{3}{4}}x^{\frac{1}{2}}\right)^2 + \left(27^{\frac{4}{3}}x^{\frac{1}{4}}\right)^4 = (3x^a)^b$, find the value of a and of b. $a = 3,\ b = 2$

Q29 Find the value of x and y when

(a) $\frac{7 \times 7^2}{\sqrt[3]{7}} = 7^x$, $x = \frac{8}{3}$

(b) $\left(\frac{1}{3}\right)^{-2} = 27^{\frac{1}{3}} + 9^y$. $y = -\frac{1}{4}$

Q30 Given $\frac{32}{11}(11^{2n}) + 11^{2n-1} = 3$, find the value of n. $n = 0$

Q31 Solve the equation $\frac{4^{x+2}}{2^{2x-1}} = \frac{8^x}{16}$. $x = 4$

Q32 Find x if $\left(\frac{8}{2^x}\right)^{-2} = 4^{4x}$. $x = \frac{3}{5}$

Q33 Solve the equation $\sqrt{(x+1)\sqrt{(x+1)\sqrt{(x+1)}}} = 2^{\frac{9}{2}}$. 34.3

Q34 If $2^x = (2^{28} - 2^{25})(2^3 - 2^1)$, find the value of x. $x = 28$

Q35 Solve $(3^x)^{x+2} = 27$. $x = -3,\ x = 1$

Q36 Solve the simultaneous equations

$2x+3y=-4$

$x-\frac{y-2x}{2}=3$

$x=1, y=-2$

Q37 Solve the simultaneous equations

$\frac{1}{2}x+\frac{1}{3}y=4$,

$\frac{1}{6}x-\frac{1}{2}y=-4\frac{1}{6}$.

$x=2,\ y=9$

Q38 Solve the simultaneous equations.

$1.2x-0.4y=0.8$

$0.25y+0.15x=0.3$

$x=\frac{8}{9}, y=\frac{2}{3}$

Q39 (a) Solve the simultaneous equations

$x+3y=10$

$2x+y=5$

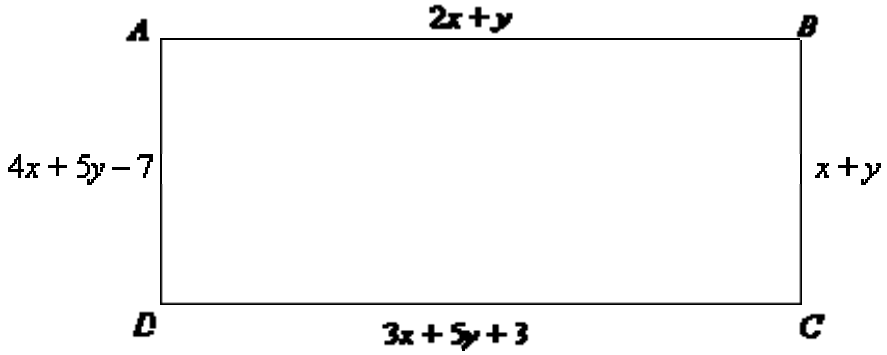
$x=1, y=3$

(b) Given that $\frac{p}{r}+\frac{3q}{r}=10, \frac{2p}{r}+\frac{q}{r}=5$ where p, q and r are non-zero numbers. Using the result of (a), find p: q: r. 1 : 3 : 1

(c) In (b), if $r = 2$, what are the values of p and q? $p = 2, q = 6$

Q40 Solve the following simultaneous equations

$5x+3y+10=7x+7y-30=8x+6y-29$. $x = 6, y = 7$



Q41 In the diagram, $ABCD$ is a rectangle. Given that $AB = (2x + y)$ cm, $BC = (x + y)$ cm, $CD = (3x + 5y + 3)$ cm and $AD = (4x + 5y – 7)$ cm.

(a) Form two equations involving x and y.

(b) Find the values of x and y. $x = 5, y = –2$

(c) Calculate the area of rectangle $ABCD$. 24 cm^2

Q42 Find the value of k for which the following simultaneous equations has no solution.

$2x-3ky=1$ and $4x+(k+2)y=5$. $k=-\frac{2}{7}$

Q43 If $(x, y) = (3, 4)$ is a solution of the simultaneous equations.

$$ax + by = 4$$
$$bx + ay = 8$$

find the value of a and of b. $a = \frac{20}{7}, b = -\frac{8}{7}$

Q44 If $(5a \quad 0)$ is divided by b, the result is 3 and the remainder is 4. If $(b + 8)$ is divided by a, the result is 3 and the remainder is $\frac{1}{4}a$. Form two equations in a and b and solve for a and b. $a = 4, b = 5$

Q45 Given that $1 \le x \le 4$ and $-8 \le y \le 5$, find

(a) the largest possible value of $y^2 - x^2$, 63

(b) the smallest possible value of $\frac{y^2}{x}$, 0

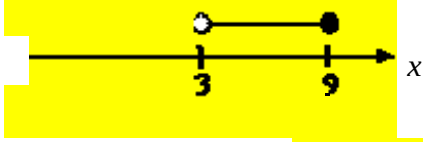
(c) the largest possible value of $x^2 - y$. 24

Q46 (a) Find the integer values of x which satisfy the inequalities
$3x + 6 \le 5x - 1 < 4x + 7$. $x = 4, 5, 6, 7$

(b) If p and q are two values of x which satisfy the inequalities in (a), find the least value of $\frac{1}{p} - \frac{p}{q}$. $-1\frac{17}{28}$

Q47 Solve the inequality $-3 < \frac{x-12}{3} \le \frac{x-15}{6}$ Show your solution on the number line.

$3 < x \le 9$



Q48 Solve the following inequality $\frac{1}{2}x + 3 < \frac{1}{3}x + 5 < x - 1$. $9 < x < 12$

Q49 Solve the inequality $\frac{2}{3}m - \frac{m-1}{4} < 7$. $m < 16\frac{1}{5}$

Q50 Find the smallest integer value of x for which $2x > 5 \times 10^9$. $x = 2500000001$