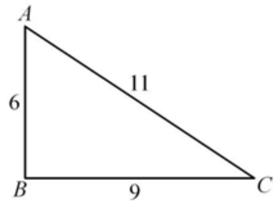
Revision Paper 10: Pythagoras' Theorem Answers

Q1 **Solution:**

(a)



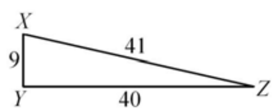
$AC^2 = 11^2$
$= 121$

$AB^2 + BC^2 = 6^2 + 9^2$
$= 117$

$\therefore AC^2 \neq AB^2 + BC^2$

Hence, ΔABC is not a right-angled triangle.

(b)



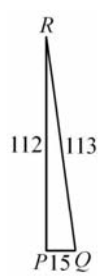
$XZ^2 = 41^2$
$= 1681$

$XY^2 + YZ^2 = 9^2 + 40^2$
$= 1681$

$\therefore XZ^2 = XY^2 + YZ^2$

Hence, ΔXYZ is a right-angled triangle.
(converse of Pythagoras' Theorem)

(c)



$RQ^2 = 113^2$

$= 12\ 769$

$PQ^2 + PR^2 = 15^2 + 112^2$

$= 12\ 769$

$\therefore RQ^2 = PQ^2 + PR^2$

Hence, ΔPQR is a right-angled triangle.

(converse of Pythagoras' Theorem)

Q2 **Solution:**

(a) $\angle B = 90°$ (given)

$AC^2 = AB^2 + BC^2$ (Pythagoras' Theorem)

$50^2 = 48^2 + BC^2$

$BC^2 = 50^2 - 48^2$

$= 196$

$\therefore BC = \sqrt{196}$

$= 14\text{ cm}$

(b) Area of $\Delta ABC = \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times 48 \times 14$

$= 336\text{ cm}^2$

Q3 **Solution:**

(a) In ΔXYZ,

$\angle YZX = 180° - 90° - 45°$

$= 45°$

$\therefore \Delta XYZ$ is an isosceles Δ.

Let $XY = YZ = w$ cm.

$\therefore w^2 + w^2 = 50^2$ (Pythagoras' Theorem)

$2w^2 = 2500$

$w^2 = 1250$

$w = \sqrt{1250}$

$= 35.4$ (correct to 3 sig. fig.)

$\therefore YZ = 35.4\text{ cm}$

(b) In ΔWXY,

$\angle WYX = 180° - 90° - 45°$

$= 45°$

$\therefore WX = WY$ (ΔWXY is an isos. Δ.)

$\angle WYZ = 90° - \angle WYX$

$= 45°$

$\angle WZY = \angle WYZ = 45°$

$\therefore WY = WZ$ (ΔWYZ is an isos. Δ)

$\therefore WX = WY = WZ$ (shown)

Q4 **Solution:**

(a) In ΔQSR,

$\angle QSR = 90°$

$QR^2 = QS^2 + RS^2$ (Pythagoras' Theorem)

$13^2 = 12^2 + RS^2$

$RS^2 = 13^2 - 12^2$

$= 25$

$\therefore RS = \sqrt{25}$

$= 5 \text{ cm}$

(b) $PS = 21 - 5$

$= 16$ cm

In ΔPQS,

$PQ^2 = QS^2 + PS^2$ (Pythagoras' Theorem)

$PQ^2 = 12^2 + 16^2$

$= 400$

$\therefore PQ = \sqrt{400}$

$= 20 \text{ cm}$

Q5 **Solution:**

(a) In ΔABD,

$\angle ABD = 90^{o}$

$AD^2 = AB^2 + BD^2$ (Pythagoras' Theorem)

$25^2 = AB^2 + 20^2$

$AB^2 = 25^2 - 20^2$

$= 225$

$\therefore AB = \sqrt{225}$

$= 15 \text{ m}$

(b) In ΔABC,

$AC^2 = AB^2 + BC^2$ (Pythagoras' Theorem)

$113^2 = 15^2 + BC^2$

$BC^2 = 113^2 - 15^2$

$= 12\ 544$

$BC = \sqrt{12\ 544}$

$= 112 \text{ m}$

$\therefore DC = 112 - 20$

$= 92 \text{ m}$

Q6 **Solution:**

(a) Area of $\Delta ABC = \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times 27 \times 16$

$= 216 \text{ cm}^2$

(b) In ΔABC,

$AC^2 = AB^2 + BC^2$ (Pythagoras' Theorem)
$AC^2 = 27^2 + 16^2$
$= 985$
$AC = \sqrt{985}$
$= 31.4$ cm (correct to 3 sig. fig.)

(c) Area of $\Delta ABC = 216\ \text{cm}^2$
$\therefore \frac{1}{2} \times AC \times BD = 216$
$\frac{1}{2} \times \sqrt{985} \times BD = 216$
$BD = \frac{432}{\sqrt{985}}$
$= 13.8$ cm (correct to 3 sig. fig.)

Q7 **Solution:**
Let the length of the other side and the hypotenuse be x cm and y cm respectively.
$\frac{1}{2} \times 13 \times x = 546$
$x = 546 \times \frac{2}{13}$
$= 84$

$y^2 = 84^2 + 13^2$ (Pythagoras' Theorem)
$= 7225$

$y = \sqrt{7225}$
$= 85$

Hence, the lengths of the other two sides are 84 cm and 85 cm.

Q8 **Solution:**
Let the length of the other side and the hypotenuse be x cm and y cm respectively.
$\therefore x + y = 180 - 18$
$x + y = 162$ ------- (1)

$y^2 = x^2 + 18^2$ (Pythagoras' Theorem)
$y^2 - x^2 = 18^2$
$(y + x)(y - x) = 324$
Substituting (1),
$162(y - x) = 324$
$y - x = 2$ -------- (2)

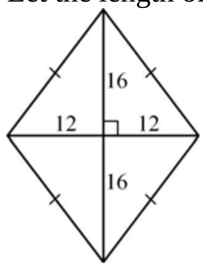
(1) + (2):
$2y = 164$
$y = 82$

Substituting $y = 82$ into (1),
$x + 82 = 162$
$x = 80$

Hence, the lengths of the other two sides are 80 cm and 82 cm.

Q9 **Solution:**

(a) Let the length of the rhombus be *h* cm.



$h^2 = 16^2 + 12^2$ (Pythagoras' Theorem)
$= 400$
$h = \sqrt{400}$
$= 20$

Hence, the length of the rhombus is 20 cm.

(b) Perimeter of rhombus $= 4 \times 20$
$= 80$ cm

Q10 **Solution:**

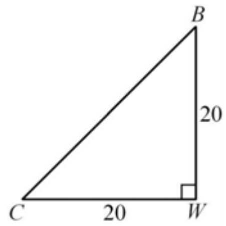
(a) $AC^2 = AB^2 + BC^2$ (Pythagoras' Theorem)
$(9x-2)^2 = (2x+1)^2 + (8x)^2$
$81x^2 - 36x + 4 = 4x^2 + 4x + 1 + 64x^2$
$13x^2 - 40x + 3 = 0$ (shown)

(b) $13x^2 - 40x + 3 = 0$
$(13x-1)(x-3) = 0$
$13x - 1 = 0$ or $x - 3 = 0$
$x = \frac{1}{13}$ or $x = 3$

(c) $x = \frac{1}{13}$ is not applicable as substituting $x = \frac{1}{13}$ into $(9x - 2)$ gives a negative value for the hypotenuse.

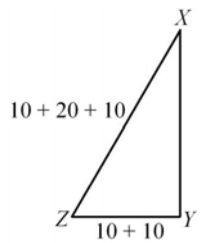
Q11 **Solution:**

(a) Let the lowest point of cylinder III be W.



$$BC^2 = CW^2 + WB^2$$
$$BC^2 = 20^2 + 20^2$$
$$= 800$$
$$BC = \sqrt{800}$$
$$= 28.3 \text{ cm (correct to 3 sig. fig.)}$$

(b) Let X, Y, Z be the centres of cylinders I, V and IV respectively.



$$XZ^2 = XY^2 + ZY^2 \text{ (Pythagoras' Theorem)}$$
$$40^2 = XY^2 + 20^2$$
$$XY^2 = 40^2 - 20^2$$
$$= 1200$$
$$XY = \sqrt{1200} \text{ cm}$$
$$\therefore OA = 10 + \sqrt{1200} + 10$$
$$= 54.6 \text{ cm (correct to 3 sig. fig.)}$$

(c) $$AD^2 = DO^2 + OA^2 \text{ (Pythagoras' Theorem)}$$
$$= 20^2 + \left(20 + \sqrt{1200}\right)^2$$
$$AD = 58.2 \text{ cm (correct to 3 sig. fig.)}$$

Q12 **Solution:**

(a) $$OB^2 = OA^2 + AB^2 \text{ (Pythagoras' Theorem)}$$
$$= \left(\sqrt{5}\right)^2 + \left(\sqrt{5}\right)^2$$
$$= 10$$
$$OC^2 = OB^2 + BC^2 \text{ (Pythagoras' Theorem)}$$
$$= 10 + \left(\sqrt{5}\right)^2$$
$$= 15$$
$$OD^2 = OC^2 + CD^2 \text{ (Pythagoras' Theorem)}$$
$$= 15 + \left(\sqrt{5}\right)^2$$
$$= 20$$

$OE^2 = OD^2 + DE^2$ (Pythagoras' Theorem)

$= 20 + \left(\sqrt{5}\right)^2$

$= 25$

$\therefore OE = \sqrt{25}$ cm

$= 5$ cm

(b) (i) Perimeter of the figure $= 5 \times \sqrt{5} + 5$

$= 16.2$ cm (correct to 3 sig. fig.)

(ii) Area of the figure

$$= \frac{1}{2} \times \sqrt{5} \times \sqrt{5} + \frac{1}{2} \times \sqrt{5} \times \sqrt{10} + \frac{1}{2} \times \sqrt{5} \times \sqrt{15} + \frac{1}{2} \times \sqrt{5} \times \sqrt{20}$$

$$= \frac{1}{2} \times \sqrt{5}\left(\sqrt{5} + \sqrt{10} + \sqrt{15} + \sqrt{20}\right)$$

$= 15.4$ cm^2 (correct to 3 sig. fig.)

Q13 **Solution:**

Let the length of the sides of the rectangle be x cm and y cm.

$x + y = 34$ ------- (1)

$xy = 240$ ------- (2)

Let the length of the diagonal of the rectangle be z cm.

$z^2 = x^2 + y^2$

$= (x + y)^2 - 2xy$

$= 34^2 - 2(240)$

$= 676$

$z = \sqrt{676}$

$= 26$

Hence, the length of the diagonal is 26 cm.

Q14 **Solution:**

(a) Radius of each circle = 5 cm

$BG^2 = 5^2 + 10^2$ (Pythagoras' Theorem)

$= 125$

$BG = \sqrt{125}$

$= 11.2$ cm (correct to 3 sig. fig.)

(b) $BF^2 = BC^2 + FC^2$ (Pythagoras' Theorem)

$= 10^2 + 15^2$

$= 325$

$BF = \sqrt{325}$

$= 18.0$ cm (correct to 3 sig. fig.)

(c) $BE^2 = 5^2 + 30^2$ (Pythagoras' Theorem)

$= 925$

$BE = \sqrt{925}$

$= 30.4$ cm (correct to 3 sig. fig.)

Q15 **Solution:**

(a) $AB = 2w$

In ΔDBC,

$DC^2 = DB^2 + BC^2$ (Pythagoras' Theorem)

$5^2 = (2w-5)^2 + w^2$

$25 = 4w^2 - 20w + 25 + w^2$

$5w^2 - 20w = 0$

$5w(w-4) = 0$

$5w = 0$ or $w - 4 = 0$

$w = 0$ (rejected) or $w = 4$

(b) (i) In ΔABC,

$AC^2 = AB^2 + BC^2$ (Pythagoras' Theorem)

$= (2w)^2 + w^2$

$= (2\times 4)^2 + 4^2$

$= 80$

$AC = \sqrt{80}$ cm

$\therefore$ perimeter of $\Delta ABC = \sqrt{80} + 4 + 8$

$= 20.9$ cm (correct to 3 sig. fig.)

(ii) Area of $\Delta ABC = \frac{1}{2} \times BC \times AB$

$= \frac{1}{2} \times w \times 2w$

$= w^2$

$= 4^2$

$= 16$ cm^2

Q16 **Solution:**

(a) Let BD be x cm.

In ΔABD,

$AB^2 = AD^2 + BD^2$ (Pythagoras' Theorem)

$226^2 = AD^2 + x^2$

$AD^2 = 226^2 - x^2$

$(264 - DC)^2 = 226^2 - x^2$ ------- (1)

In ΔBCD,

$BC^2 = DC^2 + BD^2$ (Pythagoras' Theorem)

$50^2 = DC^2 + x^2$

$DC^2 = 50^2 - x^2$ ------- (2)

(1) – (2):

$(264 - DC)^2 - DC^2 = 226^2 - 50^2$

$(264 - DC - DC)(264 - DC + DC) = 48\ 576$

$264(264 - 2DC) = 48\ 576$

$264 - 2DC = 184$

$132 - DC = 92$

$DC = 40$ cm

Substituting $DC = 40$ into (2),

$40^2 = 50^2 - x^2$

$x^2 = 50^2 - 40^2$

$= 900$

$x = \sqrt{900}$

$= 30$

$\therefore BD = 30$ cm

(b) Area of $\Delta ABC = \frac{1}{2} \times BD \times AC$

$= \frac{1}{2} \times 30 \times 264$

$= 3960$ cm^2

Q17 **Solution:**

(a) $AC^2 = AB^2 + BC^2$ (Pythagoras' Theorem)

$= 1^2 + 2^2$

$= 5$

$AC = \sqrt{5}$

$= 2.24$ cm (correct to 3 sig. fig.)

(b) $DG^2 = 1^2 + 3^2$ (Pythagoras' Theorem)

$= 10$

$DG = \sqrt{10}$

$= 3.16$ cm (correct to 3 sig. fig.)

(c) $BE^2 = 1^2 + 4^2$ (Pythagoras' Theorem)

$= 17$

$BE = \sqrt{17}$

$= 4.12$ cm (correct to 3 sig. fig.)

(d) $AF^2 = AB^2 + BF^2$ (Pythagoras' Theorem)

$= 1^2 + 5^2$

$= 26$

$AF = \sqrt{26}$

$= 5.10$ cm (correct to 3 sig. fig.)

(e) $FG^2 = GC^2 + FC^2$ (Pythagoras' Theorem)

$= 2^2 + 3^2$

$= 13$

$FG = \sqrt{13}$

$= 3.61$ cm (correct to 3 sig. fig.)

Q18 **Solution:**

(a) Area of $\Delta ABC = \frac{1}{2} \times AB \times BC$

$= \frac{1}{2}(x+5) \times BC$

$$\therefore \frac{1}{2}(x+5)\times BC = 1.5x^2 + 10x + 12.5$$
$$(x+5)\times BC = 3x^2 + 20x + 25$$
$$BC = \frac{3x^2 + 20x + 25}{x+5}$$
$$= \frac{(3x+5)(x+5)}{x+5}$$
$$= (3x+5) \text{ cm}$$

(b) $AC^2 = AB^2 + BC^2$ (Pythagoras' Theorem)
$$100 = (x+5)^2 + (3x+5)^2$$
$$100 = x^2 + 10x + 25 + 9x^2 + 30x + 25$$
$$10x^2 + 40x - 50 = 0$$
$\therefore x^2 + 4x - 5 = 0$ (shown)

(c) $x^2 + 4x - 5 = 0$
$$(x+5)(x-1) = 0$$
$x+5=0$ or $x-1=0$
$x=-5$ (rejected) or $x=1$

(d) (i) Perimeter of $\Delta ABC = x+5+3x+5+10$
$$= 4x + 20$$
$$= 4(1) + 20$$
$$= 24 \text{ cm}$$

(ii) Area of $\Delta ABC = 1.5x^2 + 10x + 12.5$
$$= 1.5(1)^2 + 10(1) + 12.5$$
$$= 24 \text{ cm}^2$$

Q19 **Solution:**

(a) In ΔABC,
$AC^2 = AB^2 + BC^2$ (Pythagoras' Theorem)
$$13^2 = (x+y)^2 + (4y)^2$$
$$13^2 = (x+x+1)^2 + [4(x+1)]^2$$
$$13^2 = (2x+1)^2 + 16(x+1)^2$$
$$4x^2 + 4x + 1 + 16x^2 + 32x + 16 = 169$$
$$20x^2 + 36x - 152 = 0$$
$$5x^2 + 9x - 38 = 0$$
$$(5x+19)(x-2) = 0$$
$5x+19=0$ or $x-2=0$
$x = -\frac{19}{5}$ (rejected) or $x=2$

$$\therefore y = x+1$$
$$= 2+1$$
$$= 3$$

$\therefore x = 2$ and $y = 3$

(b) In ΔACD,

$AD^2 = AC^2 + CD^2$ (Pythagoras' Theorem)

$AD^2 = 13^2 + (30x + 8y)^2$

$= 13^2 + [30(2) + 8(3)]^2$

$= 13^2 + 84^2$

$= 7225$

$AD = \sqrt{7225}$

$= 85$ units

(c) (i) Perimeter of quadrilateral $ABCD = x + y + 4y + 30x + 8y + 85$

$= 31x + 13y + 85$

$= 31(2) + 13(3) + 85$

$= 186$ units

(ii) Area of quadrilateral $ABCD = \frac{1}{2}(x + y)(4y) + \frac{1}{2}(13)(30x + 8y)$

$= \frac{1}{2}(5)(12) + \frac{1}{2}(13)(84)$

$= 576$ units^2

Q20 **Solution:**

(a) Trapezium

(b) $18^2 = x^2 + x^2$ (Pythagoras' Theorem)

$2x^2 = 324$

$x^2 = 162$

$x = \sqrt{162}$

$= 12.7$ (correct to 3 sig. fig.)

$y = 50 - x - 30$

$= 20 - \sqrt{162}$

$= 7.27$ (correct to 3 sig. fig.)

$z^2 = x^2 + y^2$ (Pythagoras' Theorem)

$= 162 + \left(20 - \sqrt{162}\right)^2$

$z = 14.7$ (correct to 3 sig. fig.)

(c) (i) Perimeter of the quadrilateral

$= 30 + 50 + 18 + 14.66$

$= 113$ cm (correct to 3 sig. fig.)

(ii) Area of the quadrilateral $= \frac{1}{2}(30 + 50) \times \sqrt{162}$

$= 509$ cm^2 (correct to 3 sig. fig.)