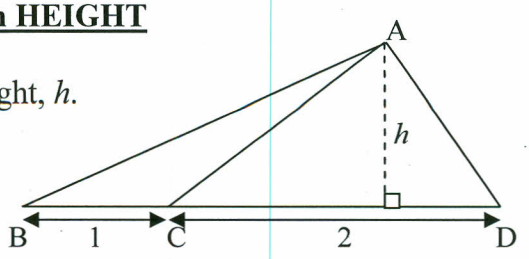
****Areas of Non-Similar Triangles**

(A) Non-Similar Triangles with a Common HEIGHT



ΔABC, ΔACD and ΔABD have the same height, *h*.

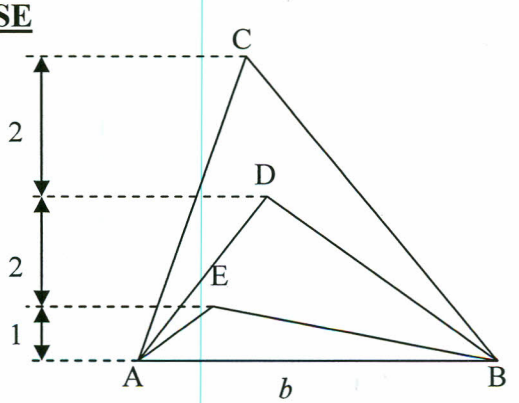
Area of ΔABC = $\frac{1}{2} \times 1 \times h$

Area of ΔACD = $\frac{1}{2} \times 2 \times h$

Area of ΔABD = $\frac{1}{2} \times 3 \times h$

∴ ratio of these areas = 1 : 2 : 3 = ratio of their base

(B) Non-Similar Triangles with a Common BASE



ΔABC, ΔABD and ΔABE have the same base, *b*.

Area of ΔABC = $\frac{1}{2} \times b \times 5$

Area of ΔABD = $\frac{1}{2} \times b \times 3$

Area of ΔABE = $\frac{1}{2} \times b \times 1$

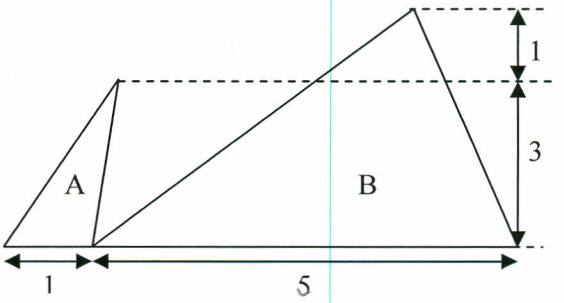
∴ ratio of these areas = 5 : 3 : 1 = ratio of their height

(C) Non-Similar Triangles without common base nor common height

Two triangles' bases are in the ratio 1 : 5 while their heights are in the ratio 3 : 4.
What is the ratio of their areas?

Area of A = $\frac{1}{2} \times 1 \times 3$

Area of B = $\frac{1}{2} \times 5 \times 4$



∴ ratio of these areas = 3 : 20 : // = ratio of their base x height

(c) ΔAQC and ΔABC have common ht

$\therefore \frac{\text{Area of } \Delta AQC}{\text{Area of } \Delta ABC} = \frac{6}{10}$

$\text{Area of } \Delta AQC = \frac{6}{10} \times 50$

$= 30 \text{ cm}^2$ ✱

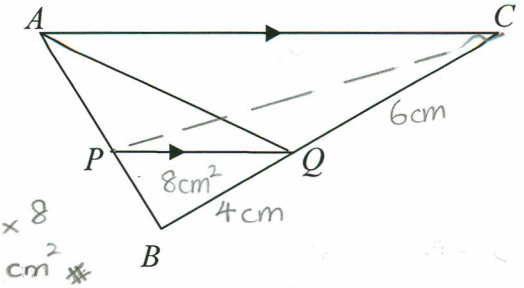
Example:

In the diagram, PQ is parallel to AC. Given that BQ = 4 cm, BC = 10 cm and area of $\Delta BPQ = 8 \text{ cm}^2$, Find the area of

(a) ΔABC;

(b) ΔPQC;

(c) ΔAQC.



(a) $\Delta ABC \sim \Delta PBQ$

$\Rightarrow \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PBQ} = \left(\frac{BC}{BQ}\right)^2$

$\text{Area of } \Delta ABC = \left(\frac{10}{4}\right)^2 \times 8$

$= 50 \text{cm}^2$ ✱

(b) ΔPQC and ΔPBQ have common height

$\therefore \frac{\text{Area of } \Delta PQC}{\text{Area of } \Delta PBQ} = \frac{6}{4}$

$\therefore \text{Area of } \Delta PQC = \frac{6}{4} \times 8$

$= 12 \text{ cm}^2$ ✱

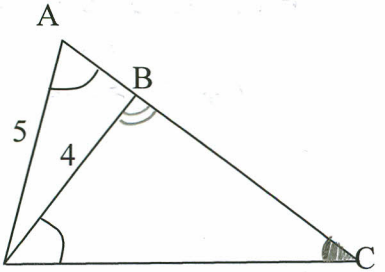
In the figure, *ABC* is a straight line and $\angle DAC = \angle BDC$.

(i) Name an angle equal to $\angle DBC$.

Given further that *AD* = 5 cm, *BD* = 4 cm and *DC* = 7 cm,

(ii) calculate *AC* and *BC*,

(iii) if the area of $\Delta BCD = a \text{ cm}^2$, find, in terms of *a*, the area of ΔABD.



(i) $\angle ADC$ ✱

(ii) $\frac{AC}{DC} = \frac{AD}{DB}$

$\frac{AC}{7} = \frac{5}{4}$

$AC = 8.75 \text{ cm}$ ✱

$\frac{BC}{DC} = \frac{DB}{AD}$

$\frac{BC}{7} = \frac{4}{5}$

$BC = 5.6 \text{ cm}$ ✱

(iii) $\Delta ADC \sim \Delta DBC$

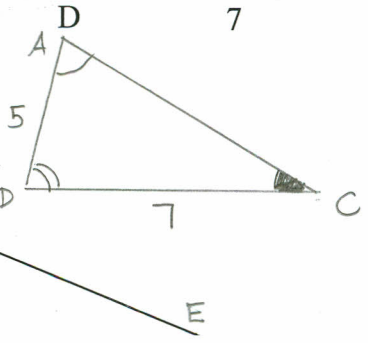
$\frac{\text{Area of } \Delta ADC}{\text{Area of } \Delta DBC} = \left(\frac{AD}{DB}\right)^2$

$\text{Area of } \Delta ADC = \left(\frac{5}{4}\right)^2 \times a$

$= \frac{25}{16} a$

$\therefore \text{Area of } \Delta ABD = \frac{25}{16} a - a$

$= \frac{9}{16} a$ ✱



Exercise

Q1 With reference to the diagram, find the value of *y*.