

## Topic: Monomials and Polynomials

### Brief Summary

An algebraic expression involves numbers, and the four basic arithmetic operational signs  $+$ ,  $-$ ,  $\times$ ,  $\div$ . The  $+$  and  $-$  signs in an algebraic expression separate it into **terms**.

A **term** is either a single number or letter or the product (or quotient) of several numbers or letters.

A **variable** can take on various values and is denoted by a letter.

A **coefficient** is a multiplying factor.

In  $3x^2 + 4x$ ,  $x$  is the **variable**. The **coefficient** of  $x^2$  is 3 and of  $x$  is 4. The **power** of  $x^2$  is 2 and that of  $x$  is 1.

A **polynomial** is an algebraic expression consisting of one or more terms, with **powers as positive integers**. It may contain one or more than one type of variables within the same expression.

**Example:**  $\frac{3}{x}$ ,  $2\sqrt{x}$  or  $3x^{-1} + 3x^{\frac{1}{2}}$  are algebraic expressions, but they are not polynomials.

$3x^2 + 4x$  is known as a polynomial in  $x$  while  $3y^2 + 4y$  is known as a polynomial in  $y$ .

$3x^2 + 4y$  is known as a polynomial in  $x$  and  $y$ .

A single term polynomial is also known as a **monomial**, e.g.  $3x^2$ ,  $4x$  etc

A polynomial with two terms is also known as a **binomial**, e.g.  $x + y$ ,  $3x^2 + 4x$ .

A polynomial with three terms is also known as a **trinomial**, e.g.  $3x^2 + 4x + 5$ .

Each polynomial has a **degree** that is given by the highest power of the variable.

$3x^2 + 4x + 5$  is a 2<sup>nd</sup> degree polynomial in  $x$ .

$3x^2 + 4x + 5$  has a value of  $3(0)^2 + 4(0) + 5 = 5$  when  $x = 0$ ,

$3x^2 + 4x + 5$  has a value of  $3(1)^2 + 4(1) + 5 = 12$  when  $x = 1$  etc

The numeral 5 always remains unchanged. This numeral is known as a constant term or simply a **constant**.

Usually, a polynomial is expressed such that the degrees of the terms appear in descending order, e.g.  $3x^2 + 4x + 5$ . Sometimes it is expressed with the degrees of the terms appearing in ascending order, e.g.  $5 + 4x + 3x^2$ .

## Topic: Monomials and Polynomials

### Extra Notes

#### (1) Addition/Subtraction of Polynomials

$x + 4$  means add 4 to the number  $x$ .

$x - 2$  means take 2 away from the number  $x$ .

$x + y$  means add the number  $x$  to the number  $y$ .

$7x$  means 7 multiplied by the number  $x$ .

$\frac{x}{5}$  means the number  $x$  divided by 5.

#### (2) Combining Polynomials

To **combine** algebraic expressions

e.g.  $2 \times t = 2t$ ,

$m \times t = mt$  (Always put the letters in alphabetical order)

$3y \times 2m = 6my$  (Always put the coefficient in front of the variables)

#### **Like terms**

Like terms only can be added or subtracted to simplify an expression

e.g.  $2x + 3x$  can be added to give  $5x$

but  $2x + 3y$  cannot give anything simpler

Expressions can be simplified by collecting like terms together.

e.g.  $2x + 3y + 3x - y$  can be simplified to  $5x + 2y$

When **brackets** occur in an algebraic expression, the rules by which operations are performed apply exactly as in arithmetic.

e.g.  $2y \times (4y - 6y) = 2y \times (-2y) = -4y^2$

#### (3) **Expansion**

A number next to a bracket indicates a multiplication, each term within the bracket must be multiplied by that number.

e.g.  $3(2t - 5) = 3 \times 2t - 3 \times 5 = 6t - 15$ ,

$[2x - 4(x - 1)] = [2x - 4x + 4] = 4 - 2x$ .

To **simplify** algebraic expression means

1. Combine **like terms**.

2. Use distributive property to remove any brackets.

Repeat step 1 and 2 until no more brackets and you can not combine anymore.