

# Indices II Solution

Date

No.

$$\begin{aligned} \text{Q1 a)} \quad & 25 - 9^{-\frac{1}{2}} + \left(\frac{1}{8}\right)^{\frac{1}{3}} \\ & = 25 - \frac{1}{9^{\frac{1}{2}}} + \sqrt[3]{\frac{1}{8}} \\ & = 25 - \frac{1}{3} + \frac{1}{2} \\ & = \frac{151}{6} \# \end{aligned}$$

$$\begin{aligned} \text{b)} \quad & \left(\frac{81}{256}\right)^{-\frac{3}{4}} = \left(\frac{256}{81}\right)^{\frac{3}{4}} \\ & = \left(\frac{4^4}{3^4}\right)^{\frac{3}{4}} \\ & = \left(\frac{4}{3}\right)^3 \\ & = \frac{64}{27} \# \end{aligned}$$

$$\begin{aligned} \text{c)} \quad & 0.001^{-\frac{2}{3}} = \left(\frac{1}{1000}\right)^{-\frac{2}{3}} \\ & = (1000)^{\frac{2}{3}} \\ & = \sqrt[3]{1000^2} \\ & = 100 \# \end{aligned}$$

$$\begin{aligned} \text{d)} \quad & (-32)^{-\frac{1}{5}} = \left(-\frac{1}{32}\right)^{\frac{1}{5}} \\ & = \left(-\frac{1}{2^5}\right)^{\frac{1}{5}} \\ & = -\frac{1}{2} \# \end{aligned}$$

$$\begin{aligned} \text{e)} \quad & \left(\frac{1}{9}\right)^{-\frac{3}{2}} = 9^{\frac{3}{2}} \\ & = \sqrt{9^3} \\ & = 27 \# \end{aligned}$$

f) same as Q1(a)

$$\begin{aligned} \text{g)} \quad & \left(\frac{81}{256}\right)^{-0.25} \times 0.0008^{-1} \\ & = \left(\frac{256}{81}\right)^{\frac{1}{4}} \times \left(\frac{8}{10000}\right)^{-1} \\ & = \left(\frac{4^4}{3^4}\right)^{\frac{1}{4}} \times \left(\frac{10000}{8}\right)^{-1} \\ & = \frac{4^1}{3} \times \frac{10000}{8^2} \\ & = \frac{5000}{3} \# \end{aligned}$$

$$\begin{aligned} \text{h)} \quad & \left(\frac{27}{8}\right)^{\frac{2}{3}} \times \left(\frac{25}{9}\right)^{-\frac{1}{2}} \times \left(\frac{3}{5}\right)^0 \\ & = \left(\frac{9}{4}\right) \times \left(\frac{3}{5}\right) \times 1 \\ & = \frac{27}{45} \# \end{aligned}$$

$$\begin{aligned} \text{i)} \quad & 3^{-\frac{1}{2}} \cdot 9^{-\frac{1}{2}} \cdot 27^{-\frac{1}{2}} \cdot 81^{\frac{3}{2}} \\ & = 3^{-\frac{1}{2}} \cdot 3^{-1} \cdot 3^{-\frac{3}{2}} \cdot 3^6 \\ & = 3^3 \\ & = 27 \# \end{aligned}$$

$$\begin{aligned} \text{j)} \quad & 16^{\frac{2}{3}} \times 6^{\frac{1}{3}} \times 3^{\frac{2}{3}} \\ & = 2^{\frac{8}{3}} \times 2^{\frac{1}{3}} \times 3^{\frac{1}{3}} \times 3^{\frac{2}{3}} \\ & = 2^3 \times 3 \\ & = 24 \# \end{aligned}$$

$$\begin{aligned} \text{k)} \quad & \frac{5^{\frac{1}{4}} \times \sqrt{10} \times \sqrt[4]{2}}{20^{\frac{3}{4}}} = \frac{5^{\frac{1}{4}} \times 2^{\frac{1}{2}} \times 5^{\frac{1}{2}} \times 2^{\frac{1}{4}}}{5^{\frac{3}{4}} \times 2^{\frac{3}{4}} \times 2^{\frac{3}{4}}} \\ & = \frac{1}{2^{\frac{3}{4}}} \# \end{aligned}$$

$$\begin{aligned} \text{l)} \quad & \sqrt[5]{14} \times \sqrt[3]{42} \div \sqrt[7]{27} \\ & = 14^{\frac{1}{5}} \times 42^{\frac{1}{3}} \div 27^{\frac{1}{7}} \\ & = 7^{\frac{1}{5}} \times 2^{\frac{1}{5}} \times 3^{\frac{1}{3}} \times 7^{\frac{1}{3}} \times 2^{\frac{1}{3}} \div 3^{\frac{3}{7}} \\ & = 7^{\frac{8}{15}} \times 2^{\frac{8}{15}} \times 3^{\frac{-2}{21}} \\ & = \frac{14^{\frac{8}{15}}}{3^{\frac{2}{21}}} \# \end{aligned}$$

$$\begin{aligned} \text{m)} \quad & \frac{\sqrt[5]{4} \sqrt{8} (\sqrt[3]{9} \sqrt{4})^2}{\sqrt[3]{12}} = \frac{4^{\frac{1}{5}} 8^{\frac{1}{2}} ((4^{\frac{1}{3}})^{\frac{1}{3}})^2}{(2^{\frac{1}{2}})^{\frac{1}{3}}} \\ & = \frac{2^{\frac{2}{5}} 2^{\frac{3}{2}} 2^{\frac{4}{15}}}{2^{\frac{1}{6}}} \\ & = 2^{\frac{12+45+8-5}{30}} \\ & = 2^2 \\ & = 4 \# \end{aligned}$$

$$\begin{aligned} \text{n)} \quad & \left(\frac{1}{125}\right)^{-\frac{1}{3}} + (-2)^{-2} + (-2)^2 + (-2)^{-2} \\ & = 125^{\frac{1}{3}} + \left(-\frac{1}{2}\right)^2 + (-2)^2 + \left(-\frac{1}{2}\right)^2 \\ & = 5 + \frac{1}{4} + 4 + \frac{1}{4} \\ & = 9\frac{1}{2} \# \end{aligned}$$

$$\begin{aligned}
 2a) \quad & (\sqrt[5]{x^2})^{\frac{1}{2}} \times (\sqrt[3]{x^5})^{\frac{1}{5}} \\
 & = (x^{\frac{2}{5}})^{\frac{1}{2}} \times (x^{\frac{5}{3}})^{\frac{1}{5}} \\
 & = x^{\frac{1}{5}} \times x^{\frac{1}{3}} \\
 & = x^{\frac{8}{15}} \#
 \end{aligned}$$

$$\begin{aligned}
 b) \quad & \frac{(-x)^4 y^{\frac{1}{3}}}{4x} \times \frac{8(-x)^{-3}}{y^{-\frac{4}{3}}} \\
 & = \frac{x^4 y^{\frac{1}{3}}}{4x} \times \frac{-8 y^{\frac{4}{3}}}{x^3} \\
 & = -2 y^{\frac{5}{3}} \#
 \end{aligned}$$

$$\begin{aligned}
 c) \quad & [(a^2)^{-\frac{1}{3}}]^4 \times [(a^4)^2]^{\frac{1}{3}} \\
 & = a^{-\frac{8}{3}} \times a^{\frac{8}{3}} \\
 & = 1 \#
 \end{aligned}$$

$$\begin{aligned}
 d) \quad & \frac{\sqrt{qr^3}}{\sqrt[4]{p^3 r^3}} \div \frac{\sqrt[4]{pr}}{\sqrt{q}} \\
 & = \frac{(qr^3)^{\frac{1}{2}}}{(p^3 r^3)^{\frac{1}{4}}} \times \frac{q^{\frac{1}{2}}}{(pr)^{\frac{1}{4}}} \\
 & = \frac{q^{\frac{1}{2}} r^{\frac{3}{2}}}{p^{\frac{3}{4}} r^{\frac{3}{4}}} \times \frac{q^{\frac{1}{2}}}{p^{\frac{1}{4}} r^{\frac{1}{4}}} \\
 & = \frac{qr^{\frac{1}{2}}}{p} \#
 \end{aligned}$$

$$\begin{aligned}
 e) \quad & (16a^4)^{\frac{1}{4}} \div (0.001a^{-3})^{\frac{1}{3}} \\
 & = \frac{1}{(16a^4)^{\frac{1}{4}}} \div 0.001 a^{-1} \\
 & = \frac{1}{2a} \div \frac{1}{1000a} \\
 & = \frac{1}{2a} \times \frac{1000a}{1} \\
 & = 500 \#
 \end{aligned}$$

$$\begin{aligned}
 f) \quad & (a^{\frac{3}{4}} \times a^{\frac{3}{8}} \div a^{\frac{3}{2}})^4 \div (a^{\frac{1}{2}} \times a^{\frac{1}{4}})^2 \\
 & = (a^{\frac{3}{2}} \times a^{\frac{3}{2}} \div a^6) \div (a^1 \times a^{\frac{1}{2}})^2 \\
 & = (a^{-\frac{3}{2}}) \div (a^{\frac{3}{2}}) \\
 & = a^{-3} \\
 & = \frac{1}{a^3} \#
 \end{aligned}$$

$$\begin{aligned}
 3a) \quad & x^{-0.5} = \frac{1}{2} \\
 & (x^{-\frac{1}{2}})^{-2} = \left(\frac{1}{2}\right)^{-2} \\
 & x = 2^2 \\
 & x = 4 \#
 \end{aligned}$$

$$\begin{aligned}
 b) \quad & 3^{2x+1} = \frac{1}{27} \\
 & 3^{2x+1} = 3^{-3} \\
 & 2x+1 = -3 \\
 & 2x = -4 \\
 & x = -2 \#
 \end{aligned}$$

$$c) \frac{2^{x-1}}{4^{1-x}} = 1$$

$$\frac{2^{x-1}}{2^{2-2x}} = 2^0$$

$$2^{3x-3} = 2^0$$

$$3x = 3$$

$$x = 1 \#$$

$$d) x^{-1.25} = \frac{243}{32}$$

$$x^{-\frac{5}{4}} = \frac{3^5}{2^5}$$

$$(x^{-\frac{5}{4}})^{-\frac{4}{5}} = \left[ \left( \frac{3}{2} \right)^5 \right]^{-\frac{4}{5}}$$

$$x = \left( \frac{3}{2} \right)^4$$

$$x = \frac{16}{81} \#$$

$$e) (3x-4)^4 = 81$$

$$(3x-4)^{4 \times \frac{1}{4}} = 81^{\frac{1}{4}}$$

$$3x-4 = 3$$

$$3x = 7$$

$$x = \frac{7}{3} \#$$

$$f) 3^{1-x} \times 9^{x+2} \times 27^{x-1} = 243$$

$$3^{1-x} \times 3^{2x+4} \times 3^{3x-3} = 3^5$$

$$3^{4x+2} = 3^5$$

$$4x+2 = 5$$

$$4x = 3$$

$$x = \frac{3}{4} \#$$

$$4a) \frac{25^{2x}}{5^{-y}} = 1 \text{ --- (1)}$$

$$4^{2x} 8^{\frac{y}{2}} = \frac{1}{16} \text{ --- (2)}$$

$$4b) 5^x = 5(25)^y \text{ --- (1)}$$

$$(8^x)^y = 32(2)^{x^2} \text{ --- (2)}$$

$$\text{Fr (1): } 5^{4x} = 5^{-y} \text{ --- (3)}$$

$$\text{Fr (2): } 2^{4x} 2^{\frac{3y}{2}} = 2^{-4}$$

$$4x + \frac{3y}{2} = -4$$

$$\Rightarrow 8x + 3y = -8 \text{ --- (4)}$$

$$\text{Fr (3): } 4x = -y \text{ --- (5)}$$

$$\text{(5) } \times 2: 8x + 2y = 0 \text{ --- (6)}$$

$$\text{(4) - (6): } y = -8$$

$$\text{sub into (5): } 4x = 8$$

$$x = 2$$

$$\therefore x = 2, y = -8 \#$$

$$\text{Fr (1): } 5^x = 5 \cdot 5^{2y}$$

$$\therefore x = 1 + 2y \text{ --- (3)}$$

$$\text{Fr (2): } 2^{3xy} = 2^5 \cdot 2^{x^2}$$

$$3xy = 5 + x^2 \text{ --- (4)}$$

$$\text{Sub (3) into (4):}$$

$$3(1+2y)y = 5 + (1+2y)^2$$

$$3y + 6y^2 = 5 + 1 + 4y + 4y^2$$

$$2y^2 - y - 6 = 0$$

$$(2y+3)(y-2) = 0$$

$$y = 2 \text{ or } -\frac{3}{2}$$

$$\text{when } y = 2, x = 1 + 2(2)$$

$$x = 5 \#$$

$$\text{when } y = -\frac{3}{2}, x = 1 + 2\left(-\frac{3}{2}\right)$$

$$x = -2 \# \text{ DURABLE}$$



$$Q5) \quad (64^{\frac{2}{3}} x^{\frac{3}{2}})^5 \div (16^{\frac{5}{4}} x^{\frac{2}{3}})^3 = (2x^m)^n$$

$$\begin{aligned} \text{LHS} &: (2^4 x^{\frac{3}{2}})^5 \div (2^5 x^{\frac{2}{3}})^3 \\ &= 2^{20} x^{\frac{15}{2}} \div 2^{15} x^2 \\ &= 2^5 x^{\frac{11}{2}} \\ &= (2x^m)^5 \end{aligned}$$

$$\therefore n=5, \quad 5m = \frac{11}{2}$$
$$m = \frac{11}{10} *$$