Topic: Indices

Introduction

When n is a positive integer, a^n is defined as: $\underbrace{a \times a \times a \times a \times \cdots \times a}_{n \text{ times}}$ where a is called the **base** and n is called the index or exponent or **power**.

Laws of Indices

(1) $a^m \times a^n = a^{m+n}$ — Multiply same base → add power

(2) $a^m \div a^n = a^{m-n}$ — Divide same base → minus power

(3) $(a^m)^n = a^{mn}$ — Multiply power

(4) $a^m \times b^m = (ab)^m$

(5) $\dfrac{a^m}{b^m} = \left(\dfrac{a}{b}\right)^m$ — Combine if same power

(6) $a^0 = 1, \quad a \neq 0$ — Power 0 → 1 unless $a = 0$ → undefined

(7) $a^{-m} = \dfrac{1}{a^m}$ — Change sign of power

(8) $a^{\frac{1}{n}} = \sqrt[n]{a}$

$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$ — Change between index and radical/surd form

Proofs:

(1) $a^m \times a^n = a^{m+n}$

$$a^2 \times a^3 = (a \times a) \times (a \times a \times a)$$
$$= a^5 = a^{2+3}$$

(2) $a^m \div a^n = a^{m-n}$

$$a^5 \div a^3 = \frac{\cancel{a} \times \cancel{a} \times \cancel{a} \times a \times a}{\cancel{a} \times \cancel{a} \times \cancel{a}}$$
$$= a^2 = a^{5-3}$$

---

(3) $(a^m)^n = a^{mn}$

$$(a^2)^3 = (a \times a) \times (a \times a) \times (a \times a)$$
$$= a^6 = a^{2\times 3}$$

---

(4) $a^m \times b^m = (ab)^m$

$$a^3 \times b^3$$
$$= a \times a \times a \times b \times b \times b$$
$$= (a \times b)(a \times b)(a \times b)$$
$$= (a \times b)^3$$

---

(5) $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

$$a^3 \div b^3$$
$$= \frac{a \times a \times a}{b \times b \times b}$$
$$= (\frac{a}{b})(\frac{a}{b})(\frac{a}{b})$$
$$= (\frac{a}{b})^3$$

(6) $a^0 = 1, \quad a \neq 0$

$$\frac{a^4}{a^4} = a^{4-4} = a^0$$

$$\frac{\cancel{a^4}}{\cancel{a^4}} = 1$$

$$\Big\} \; a^0 = 1$$

---

(7) $a^1 \div a^3$

$$a^1 \div a^3 = \frac{a}{a \times a \times a} = \frac{1}{a^2}$$

$$a^1 \div a^3 = a^{1-3} = a^{-2}$$

$$\Big\} \; a^{-2} = \frac{1}{a^2}$$

Note: (I) $a^{-1} = \frac{1}{a}$ (II) $\frac{a^{-3}}{b^4} = \frac{b^{-4}}{a^3}$ (III) $5a^{-2} = \frac{5}{a^2}$

---

(8) $a^{\frac{1}{n}} = \sqrt[n]{a}$

$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

$$(a^{\frac{1}{3}})^3 = a$$

$$a^{\frac{1}{3}} = \sqrt[3]{a^1}$$

Note: (I) $4^{\frac{1}{2}} = \sqrt{4}$ (II) $27^{\frac{1}{3}} = \sqrt[3]{27}$ (III) $2^{\frac{3}{2}} = \sqrt{2^3} = \left(\sqrt{2}\right)^3$

Index Form Power/index

base $\longrightarrow 2^{\frac{32}{17}}$

Radical/Surd Form

$$\sqrt[17]{2^{32}}$$