Topic: Algebraic Manipulation Worksheet 1

Things to note:

1. In simplifying algebraic fractions, multiply or divide the numerator and denominator by the same non-zero number or expression.
2. Factorise the numerator and denominator to see if there are any common factors.
3. **Never cancel individual terms of the numerator and the denominator before factorisation is done.**
4. Apply rules of indices carefully.

Useful Shortcuts (Identities)

1. $a - b = -(b - a)$
2. $-a - b = -(a + b)$
3. $(a + b)(a - b) = a^2 - b^2$
4. $(a + b)^2 = a^2 + 2ab + b^2$
5. $(a - b)^2 = a^2 - 2ab + b^2$

(1) Simplifying Algebraic Fractions

This concept is similar to the arithmetical method of reducing numeral fractions to its lowest terms.

Example 1: Simplify $\frac{3x^3yz^8}{27xy^3z^4}$

Solution:

Example 2: Simplify $\frac{9(x+y)^{n+1}}{24(x+y)^{n+3}}$

Solution:

Example 3: Simplify $\frac{x^2+xy-xz-yz}{x^2+xy+xz+yz}$

Solution:

Example 4: Simplify $\frac{2a^2-17a+21}{-a^2+6a+7}$

Solution:

(2) Multiplying and Dividing Algebraic Fractions

How do you multiply fractions?

$$\frac{18}{49}\times\frac{35}{27}=\frac{9\times 2}{7\times 7}\times\frac{5\times 7}{3\times 9}=\frac{2\times 5}{7\times 3}=\frac{10}{21}$$

What did we do to 18, 35, 49 and 27? _____

We apply the same concept to algebraic fractions!

Example 1: Simplify $\frac{x^2+xy}{x^2-xy}\times\frac{xy^2+y^3}{x^3+x^2y}$

Solution:

Example 2: Simplify $\frac{x-y}{b-a}\times\frac{a-b}{y-x}$

Solution:

Things to note:

1. **To divide by a fraction, multiply by its reciprocal.**
2. **Simplify the result by cancelling common factors.**

Useful Shortcut

$$\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{ad}{bc}$$

(3) Addition and Subtraction of Algebraic Fractions

How do you add the following fractions: $\frac{2}{5}+\frac{5}{12}$?

We will use the same idea to add algebraic fractions.

Example 1: Simplify $\frac{y+2}{3}-\frac{3-y}{5}$

Solution:

Example 2: Simplify $\frac{2x}{x-1}-\frac{3x-1}{x+2}$

Solution:

Example 3: Simplify $\frac{x}{x+1}-\frac{2}{1-x^2}$

Solution:

(4) Solve equations involving algebraic fractions

Example 1: Solve the equation $\frac{2x-3}{3}=\frac{x-2}{5}$

Solution:

Example 2: Solve the equation $\frac{3x-1}{5}-\frac{2x-5}{2}=1$

Solution:

Example 3:

Solve the equation $\frac{2}{a}-3=2a$

Solution: