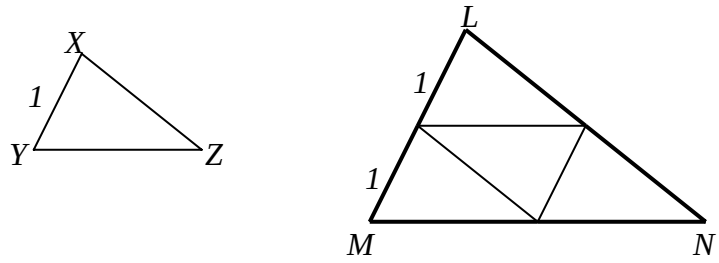
Areas of Similar Figures

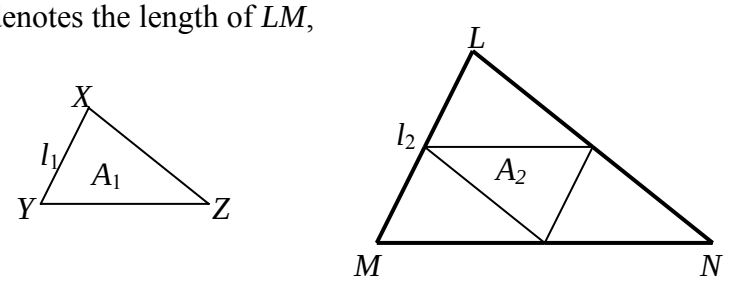
Consider the similar figures below:



Ratio of XY : LM = 1: 2

Ratio of area of ΔXYZ : area of ΔLMN = 1: 4

Hence, if A_1 denotes the area of ΔXYZ, A_2 denotes the area of ΔLMN, l_1 denotes the length of XY and l_2 denotes the length of LM,



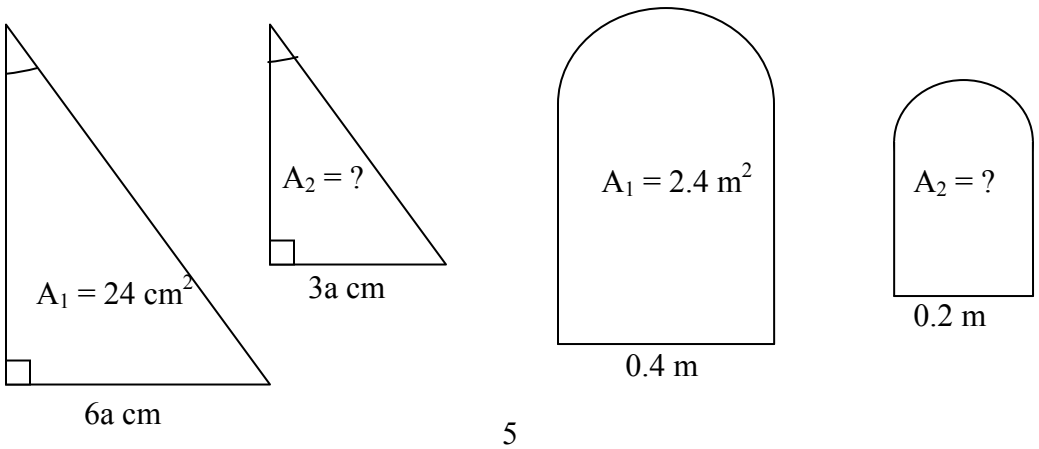
Derive the relationship between the ratio of the areas of similar figures and their corresponding lengths.

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$

In words, it means that ratio of the areas of any 2 similar figures is equal to the ratio of their lengths squared.

Examples

1 Find the unknown areas. In each case, the shapes are similar.



Ans:

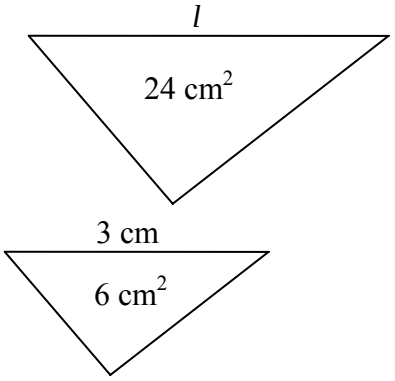
$$\frac{A_2}{A_1}=\left(\frac{l_2}{l_1}\right)^2 \qquad \frac{A_2}{A_1}=\left(\frac{l_2}{l_1}\right)^2$$

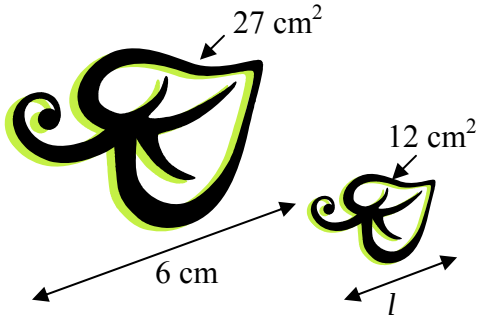
$$\frac{A_2}{24}=\left(\frac{3a}{6a}\right)^2 \qquad \frac{A_2}{2.4}=\left(\frac{0.2}{0.4}\right)^2$$

$$\frac{A_2}{24}=\frac{1}{4} \qquad \frac{A_2}{2.4}=\frac{1}{4}$$

$$A_2=6\text{ cm}^2 \qquad A_2=0.6\text{ cm}^2$$

2 Find the unknown lengths for each pair of similar shapes.





Ans:

$$\left(\frac{l_1}{l_2}\right)^2=\frac{A_1}{A_2} \qquad \left(\frac{l_1}{l_2}\right)^2=\frac{A_1}{A_2}$$

$$\left(\frac{l_1}{3}\right)^2=\frac{24}{6} \qquad \left(\frac{l_1}{6}\right)^2=\frac{12}{27}$$

$$\frac{l_1^2}{9}=4 \qquad \frac{l_1^2}{36}=\frac{12}{27}$$

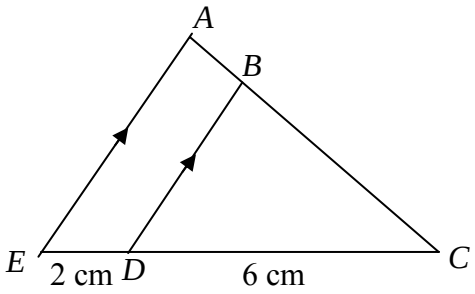
$$l_1^2=36 \qquad l_1^2=16$$

$$l_1=6\text{ cm} \qquad l_1=4\text{ cm}$$

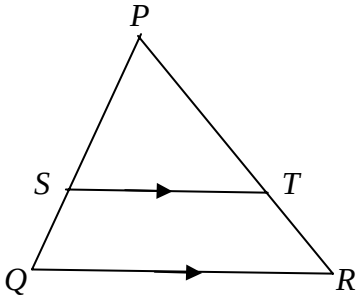
Exercises:

Q1 In a plan of a house, the width, 150 cm, of a door is represented by a line 30 mm long. Find the area of the house is the corresponding area on the plan is 3 250 cm^2.

Q2 In the figure, BD is parallel to AE, $ED = 2$ cm and $DC = 6$ cm. Given that the area of $\Delta CBD = 9$ cm^2, calculate the area of $ABDE$.



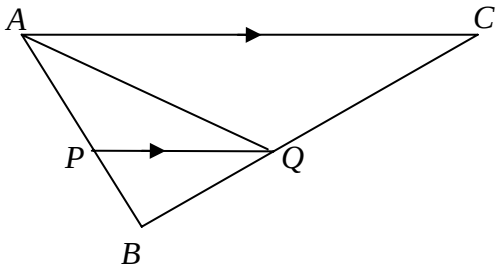
Q3 In the diagram, $PT = 6$ cm, $PR = 10$ cm and the area of $\Delta CBD = 24$ cm^2. Find
(a) the area of ΔPQR.
(b) the area of SQRT.



Q4 The radius of one sphere is $3\frac{1}{2}$ times that of another. Given that the surface area of the smaller sphere is 64 cm^2, find the surface area of the larger sphere.

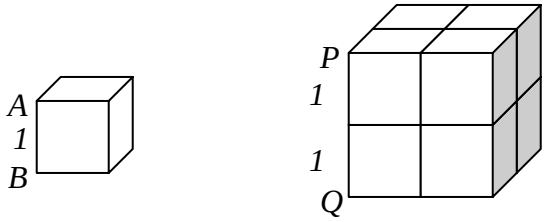
Q5 Two prisms are geometrically similar and the height one is $2\frac{1}{2}$ times that of the other.
(a) Write down the ratio of the total surface area of the smaller prism to that of the larger prism.
(b) Given that the total surface area of the larger prism is 625 cm^2, calculate the area of the smaller prism.

Q6* In the diagram, PQ is parallel to AC. Given that BQ = 4 cm, BC = 10 cm and area of $\Delta BPQ = 8$ cm^2, find
(a) the area of ΔABC;
(b) the area of ΔPQC;
(c) the area of ΔAQC.



Volume of Similar Solids

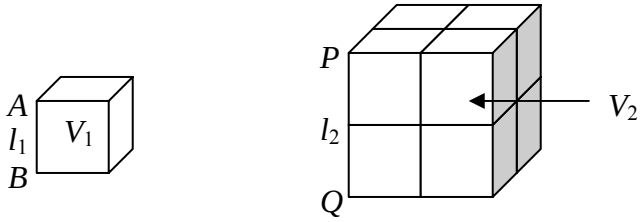
Consider the following solids:



Ratio of AB : PQ = 1 : 2

Ratio of volume of smaller solid : volume of larger solid = 1 : 8

Hence, if V_1 denotes the volume of smaller solid, V_2 denotes the volume of larger solid, l_1 denotes the length of AB and l_2 denotes the length of PQ,



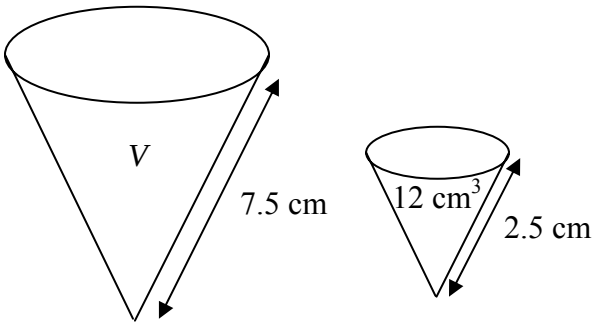
Derive the relationship between the ratio of the volume of similar solids and their corresponding lengths.

$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$

In words, it means that ratio of the volume of any 2 similar solids is equal to the ratio of their lengths cubed.

Examples:

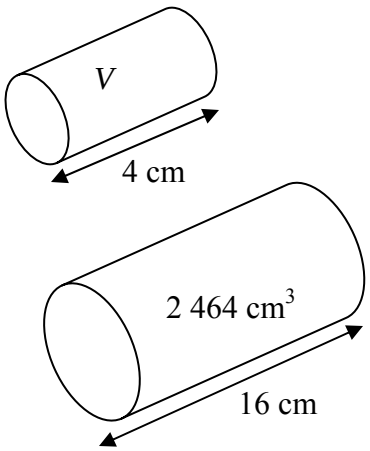
1 Find the unknown volume, V, of each of the following solids.



$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$

Ans: $$\frac{V_1}{12} = \left(\frac{7.5}{2.5}\right)^3$$

$$V_1 = 324 \text{ cm}^3$$

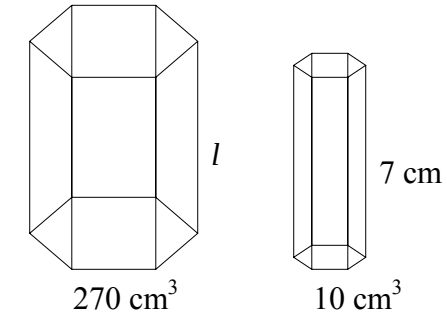


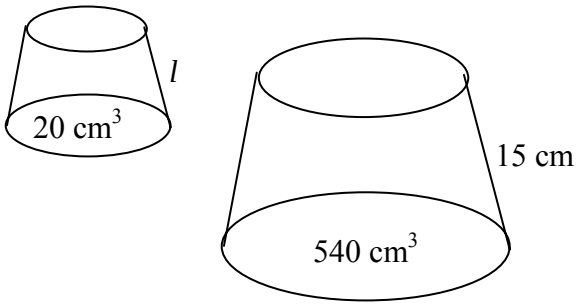
$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$

Ans: $$\frac{V_1}{2464} = \left(\frac{4}{16}\right)^3$$

$$V_1 = 38.5 \text{ cm}^3$$

2 Find the unknown length, l, in each of the following pairs of similar objects.





3 The areas of the bases of 2 similar cones are in the ratio 9:16.

(a) Find the ratio of the heights of the cones.

(b) Given that the volume of the larger cone is 448 cm^3, find the volume of the smaller cone.

Exercises:

Q1 Find the ratio of (a) the surface areas (b) the volumes of

(i) 2 similar solid cylinders of circumferences 10 cm and 8 cm;
(ii) 2 similar solid cones of heights 9 cm and 12 cm;
(iii) 2 spheres of radii 4 cm and 6 cm.

Q2 The volume of one sphere is 4 times the volume of a second sphere. If the radius of the smaller sphere is 3 cm, what is the radius of the larger sphere?

Q3 The masses of 2 similar marble toys of the same density are 8.58 kg and 4.29 kg. If the first toy is 12.94 cm high, what is the height of the second toy?

Q4 The height of the mast of the model of a boat is 14 cm, and the height of the mast of the actual boat is 2.1m.

(a) Given that the length of the model boat is 160 cm, calculate the length of the actual boat in cm.

(b) If it costs \$4 to paint the model, what will it cost to paint the actual boat?

Q5 Alan buys two bottles of milk at Golden Storage Supermarket. The bottles are geometrically similar to each other. The base diameter of one bottle is 8 cm and the base radius of the other bottle is 3 cm.

(a) Calculate the height of the small bottle if the height of the larger bottle is 28 cm.
(b) Calculate the ratio of the volume of large bottle to the volume of small bottle.
(c) The cost of the small bottle of milk is \$2.70. Calculate the cost of the large bottle of milk .

Q6 (a) Two similar solids have surface areas of 160 cm^2 and 78.4 cm^2 respectively.
(i) Write down the ratio of the heights of the solids, in the form 1 : n, where n is a constant.
(ii) If the height of the smaller solid is 6 cm, find the height of the larger solid.
(iii) Given that the volume of the larger solid is 650 cm^3, calculate the volume of the smaller solid.

(b) Two similar cones have surface areas A_1 and A_2 such that $A_1 = 169A_2$. If their volumes, V_1 and V_2 respectively, are such that $V_1 = kV_2$, write down the value of k.